**Cluster and Mini Cluster Tree Elimination**

**CTEf and MCTEf**

**Function Filtering**

**A Filtering Tree decomposition**

**IMCTE**

**Conclusions**

- Main idea: Delete tuples that will become inconsistent in the future.
- Filtering is a way of using other functions of other clusters, parts of functions of other clusters, sums and approximations of functions of other clusters.
- So is a way of going behind the "exactly one" imposition of the tree decomposition definition.
- Allows to use Upper Bounds and Lower Bounds to delete tuples.
- An elegant extension IMCTE uses the messages computed in previous iterations to delete tuples.
- Memory storage is reduced significantly.

**Search**

- Main step: guessing
- Bottleneck: exponential search tree traversal
- Time Complexity: \(O(d^k)\)
- Space Complexity: \(O(n, d)\)
- Average Time Complexity: better than worst case

**Inference**

- Main step: message passing
- Bottleneck: memory storage
- Time Complexity: \(O(v, \exp(w))\) \(v =\) tree width
- Space Complexity: \(O(v, \exp(w))\)
- Average Space Complexity: close to worst

**WCP and Valuation Structures**

**Definition 1** A valuation structure \(S(k) = \{(x_1, \ldots, x_k, \emptyset)\}\) where:

- \(k \in \{1, \ldots, \infty\}\)
- \(x \uplus b = \min\{x, a + b\}\), and
- \(\emptyset\) is the standard order among naturals.

**Definition 2** A Weighted CSP (WCP) is \((X, D, C, S)\) where:

- \(X = \{x_1, \ldots, x_k\}\)
- \(D = \{D_1, \ldots, D_k\}\)
- \(C\) is a finite set of cost functions:
- \(C\) is the standard order among naturals.

**Property 1**

\[ f(t) = \begin{cases} \emptyset & \text{if } t \text{ is allowed} \\ \emptyset \cup \{b(t)\} & \text{if } t \text{ is partially allowed} \\ \emptyset & \text{if } t \text{ is totally forbidden} \\ \end{cases} \]

**Property 2**

\[ f(t) = \emptyset \cup \{b(t)\} \]

**CTE and MCTEf**

- CTE solves WCP by sending mgs \(m(u, v)\) along the edges of a tree decomposition.
- CTE time and space complexity are \(O(v, \exp(w))\).
- \(\text{MCTEf}(e)\) approximates CTE limiting the arity: \(M(u, v) \leq m(u, v)\)

**IMCPE**

- \(\phi(u, v) = \mathcal{M}(u, v)\) the approximated IMCE message of a previous execution (one with minor \(r\))
  
**Experiments**

1. **MCTEf** sends \(m(u, v)\) with functions in \(\phi(u, v)\).

**IMCPE**

- **Operation on Functions**
  
**IMCPE**

- **Procedure IMCPE\((X, D, C, S)\)**
  
**Experiments**

1. Showing that CTE versus state of the art CTE use less tuples to find the exact solution.
2. Inside an approximation schema we show that MCTEf\((e)\) reduces resources at a smaller \(r\) and finds worst LB than the iterative version IMCE where the previous messages of MCTEf\((e)\) execution are used as filters.